

# Generalized Lifting for Lossy Image Coding

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# Outline

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- ✓ Introduction
  - Motivation and previous work
- ✓ Generalized Lifting
  - Classical Lifting
  - Generalized Lifting
  - Mapping design
- ✓ Evaluation of the potential: The ideal case
- ✓ Image coding implementations
  - Case I: Modeling of contours (global pdf estimation)
  - Case II: Adaptive local pdf estimation
- ✓ Discussion
  - Entropy coding
- ✓ Conclusion

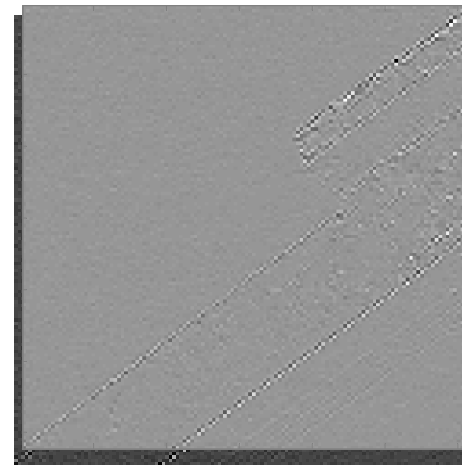
# Motivation

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- ✓ Limitations of wavelets in image representation
  - Excellent for decorrelation of impulsional-like events
  - Limitations on images because wavelets are not able to fully decorrelate edges



*DWT*



LH subband zoom

Need of additional decorrelation for coding applications

# Previous work

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## Main approaches

- Go beyond wavelets trying to overcome their limitations
- Edges become 1-dimensional localizable singularities
- Increased sparsity of the coefficients



## Frequency-domain methods

### **Curvelets**

[Candès, Donoho 1999; Candès 2006]

### **Contourlets**

[Do, Vetterli 2005]



## Spatial-domain methods

### **Bandelets**

[LePennec, Mallat 2005; Peyré, Mallat 2005]

### **Adaptive directional lifting**

[Chappelier *et al* 2006; Ding *et al* 2007; Heijmans *et al* 2005; Mehrseresht, Taubman 2006; Piella *et al* 2002, 2006; Wang *et al* 2006; Zhang *et al* 2005]

### **Generalized lifting**

May be highly non-linear,  
preserving perfect reconstruction

[Solé, Salembier 2004, 2007]

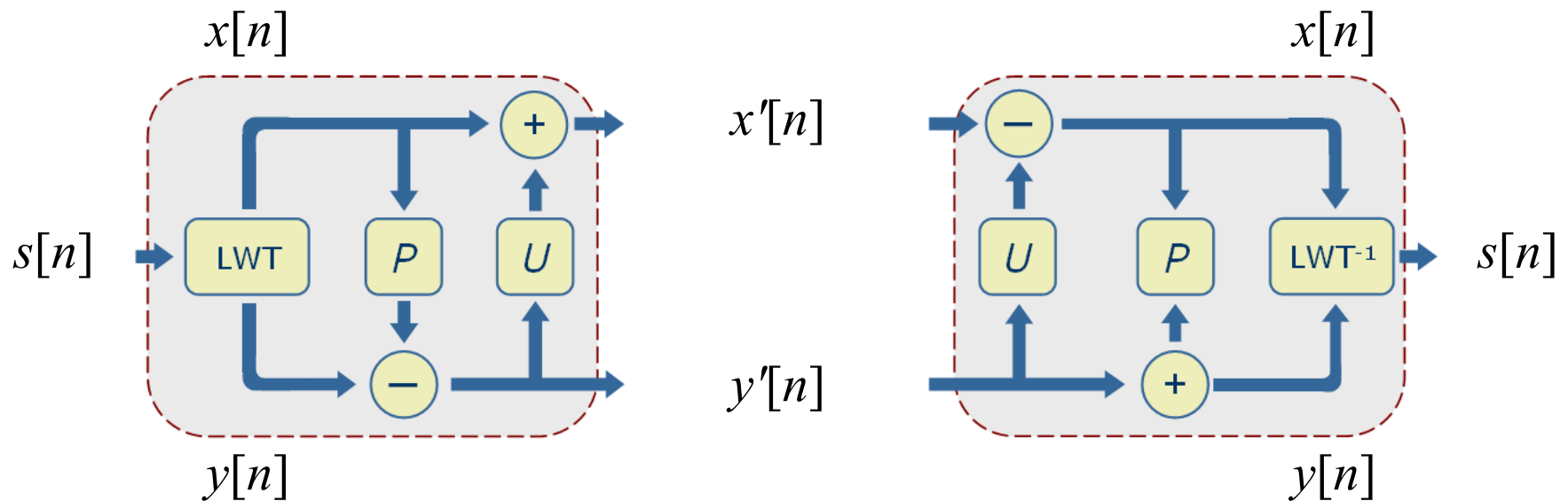
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# Classical Lifting

*Approximation signal:*  $x'[n] = x[n] + U(y'[n])$

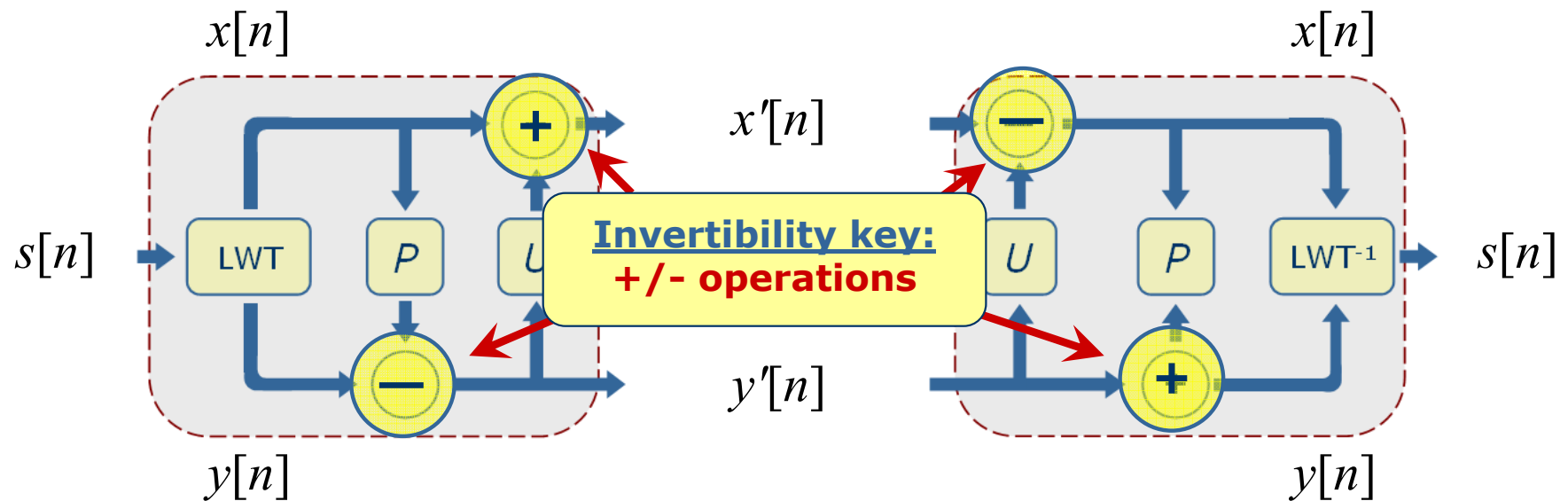


*Detail signal:*  $y'[n] = y[n] - P(x[n])$

[Sweldens 1996]

# Classical Lifting

*Approximation signal:*  $x'[n] = x[n] + U(y'[n])$

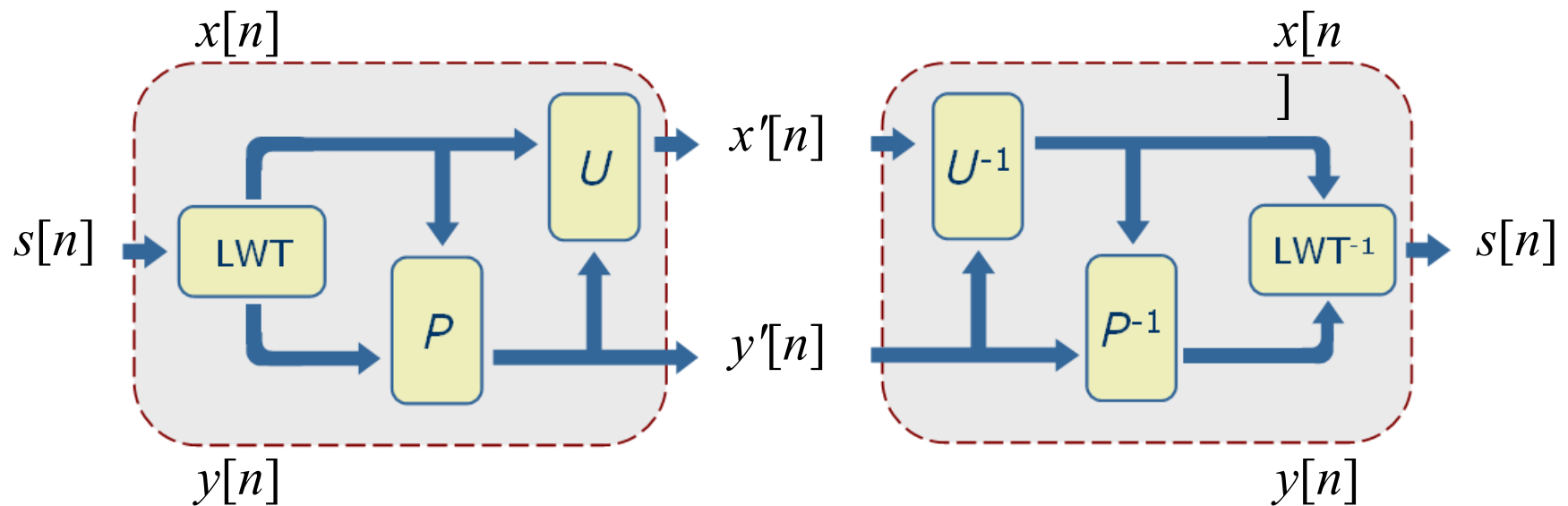


*Detail signal:*  $y'[n] = y[n] - P(x[n])$

[Sweldens 1996]

# Generalized Lifting

*Approximation signal:*  $x'[n] = U(x[n], y'[n])$

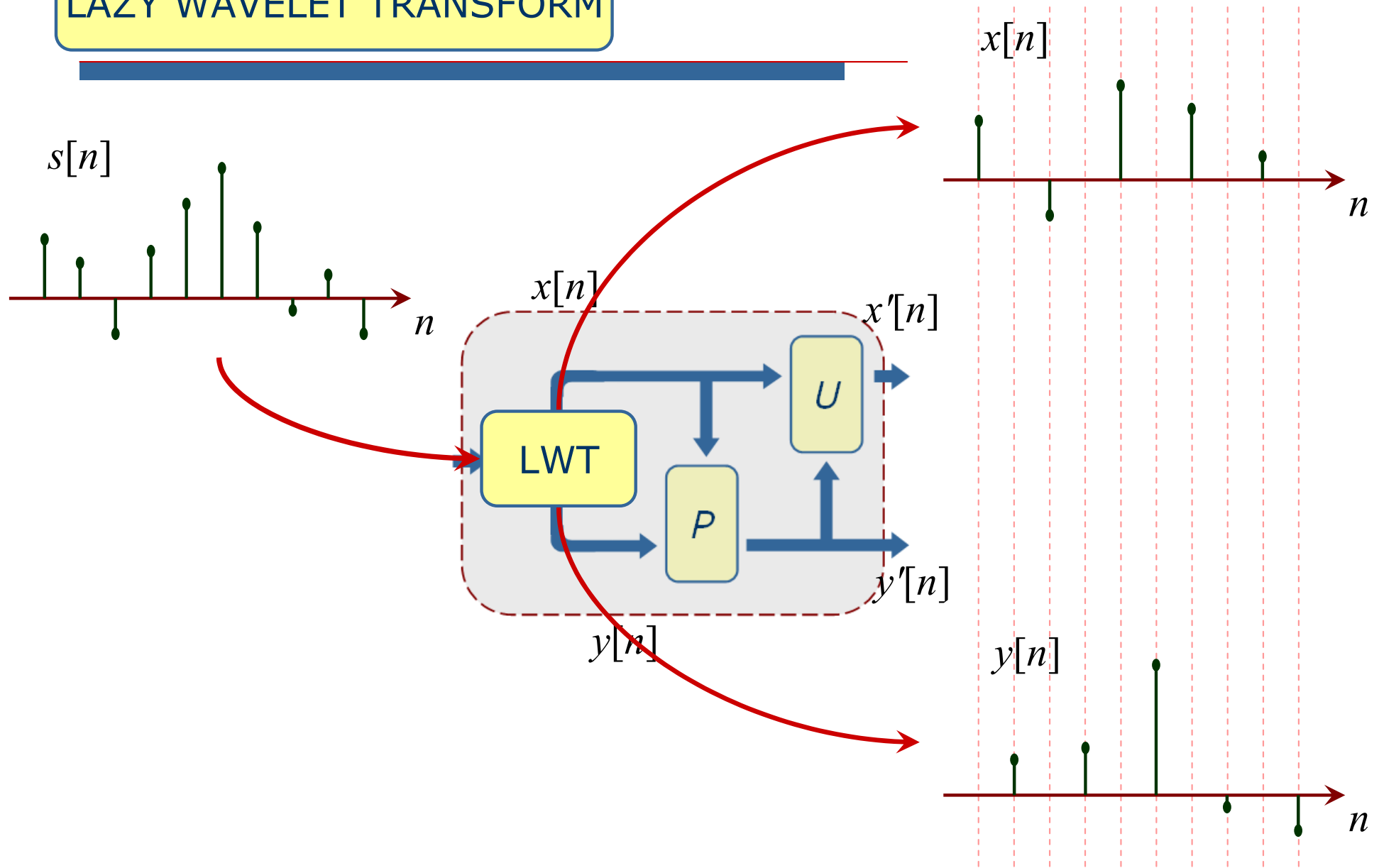


*Detail signal:*  $y'[n] = P(y[n], x[n - i])|_{i \in C}$

[Solé, Salembier 2004]

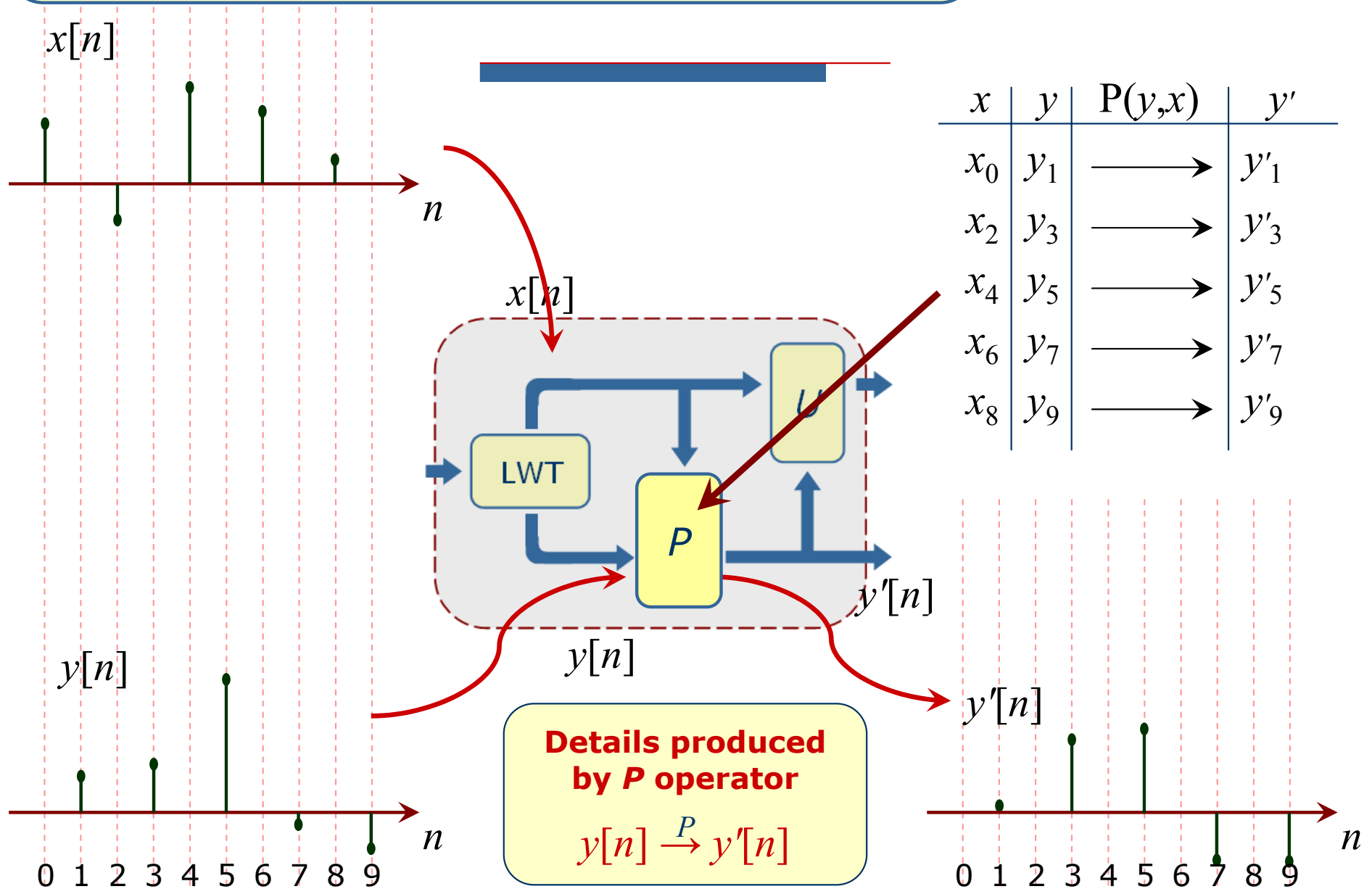


# LAZY WAVELET TRANSFORM



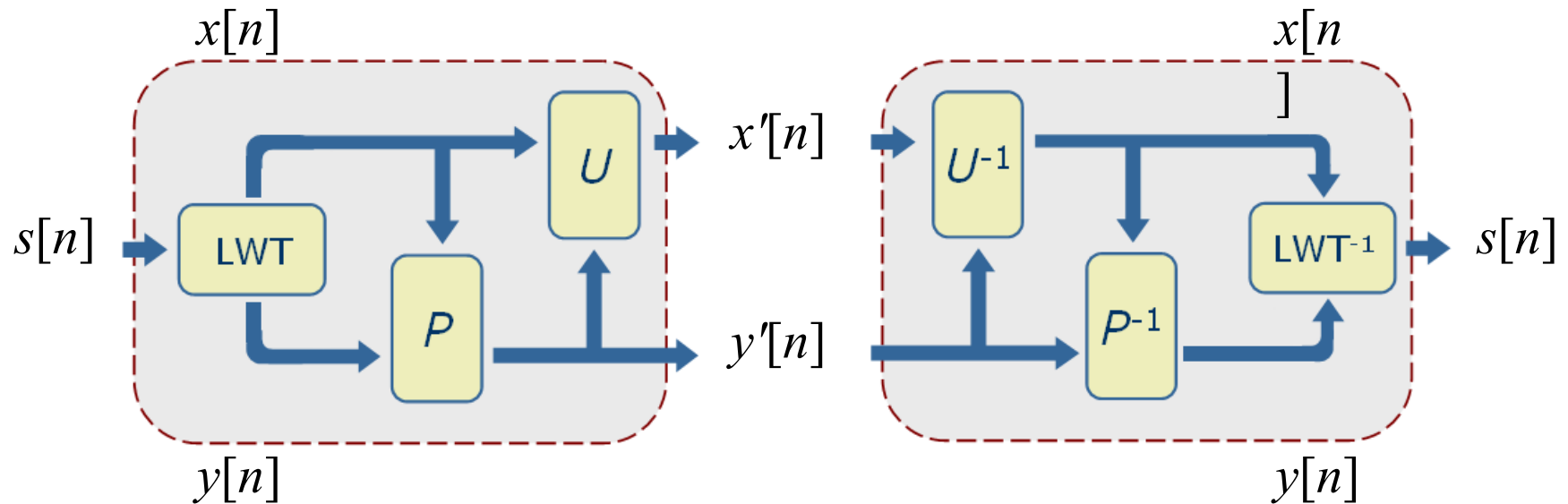
Detail signal:

$$y'[n] = P(y[n], x[n - i])|_{i \in C}$$



# Generalized Lifting

*Approximation signal:*  $x'[n] = U(x[n], y'[n])$

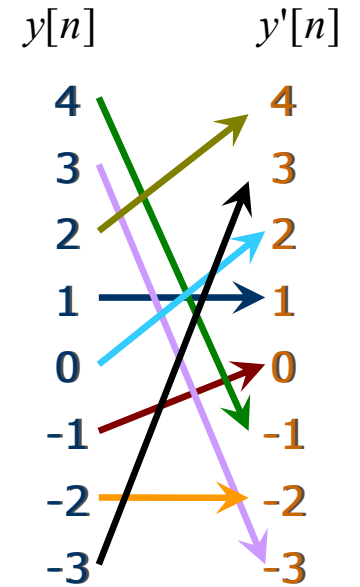


**BIJECTIVITY OF  $P, U$  IS MANDATORY  
TO ACHIEVE PERFECT RECONSTRUCTION**

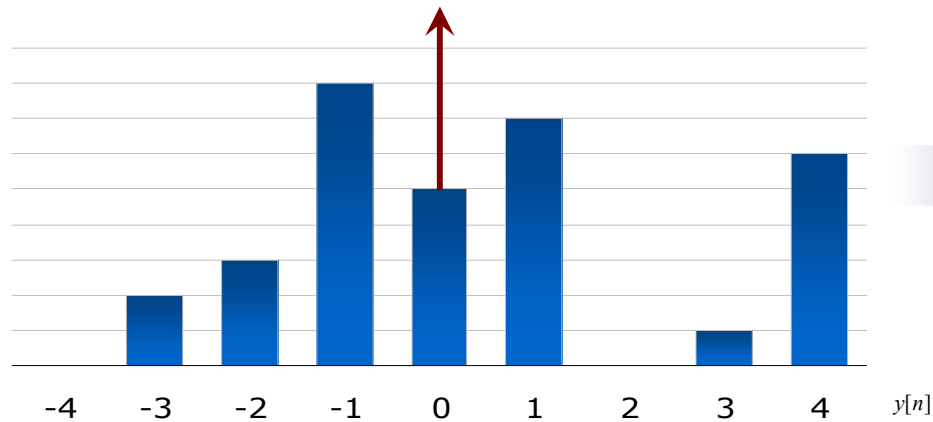
[Solé, Salembier 2004]

# Mapping design

**Criterion: Energy minimization of the mapped details**

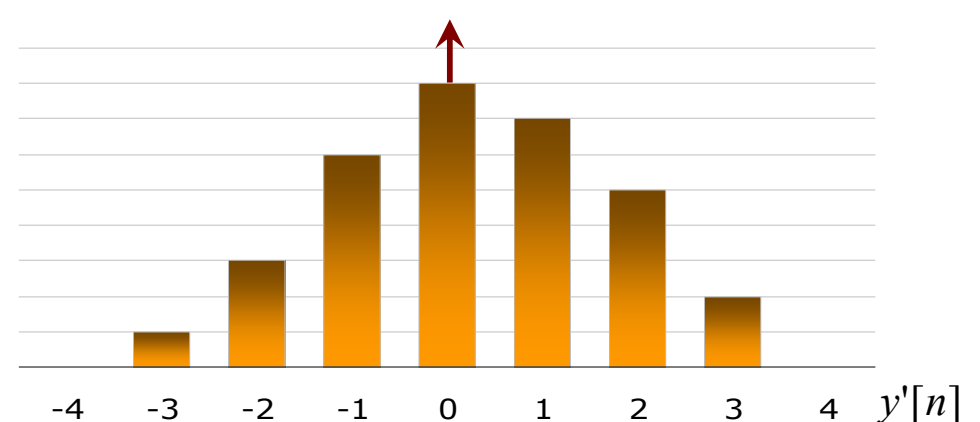


*Input signal pdf*



GL

*mapped pdf*



# Outline

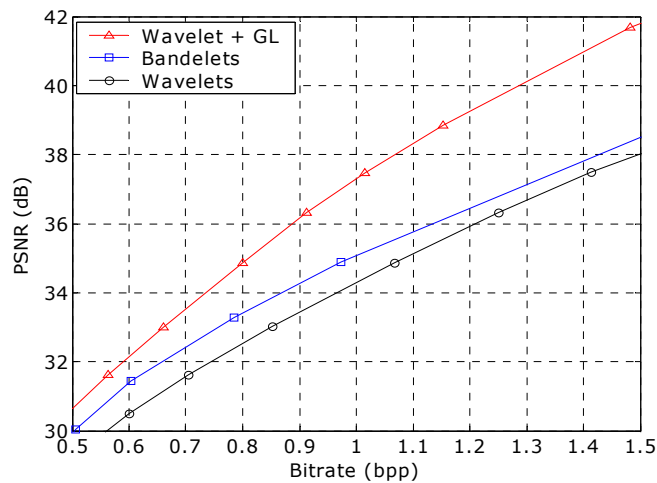
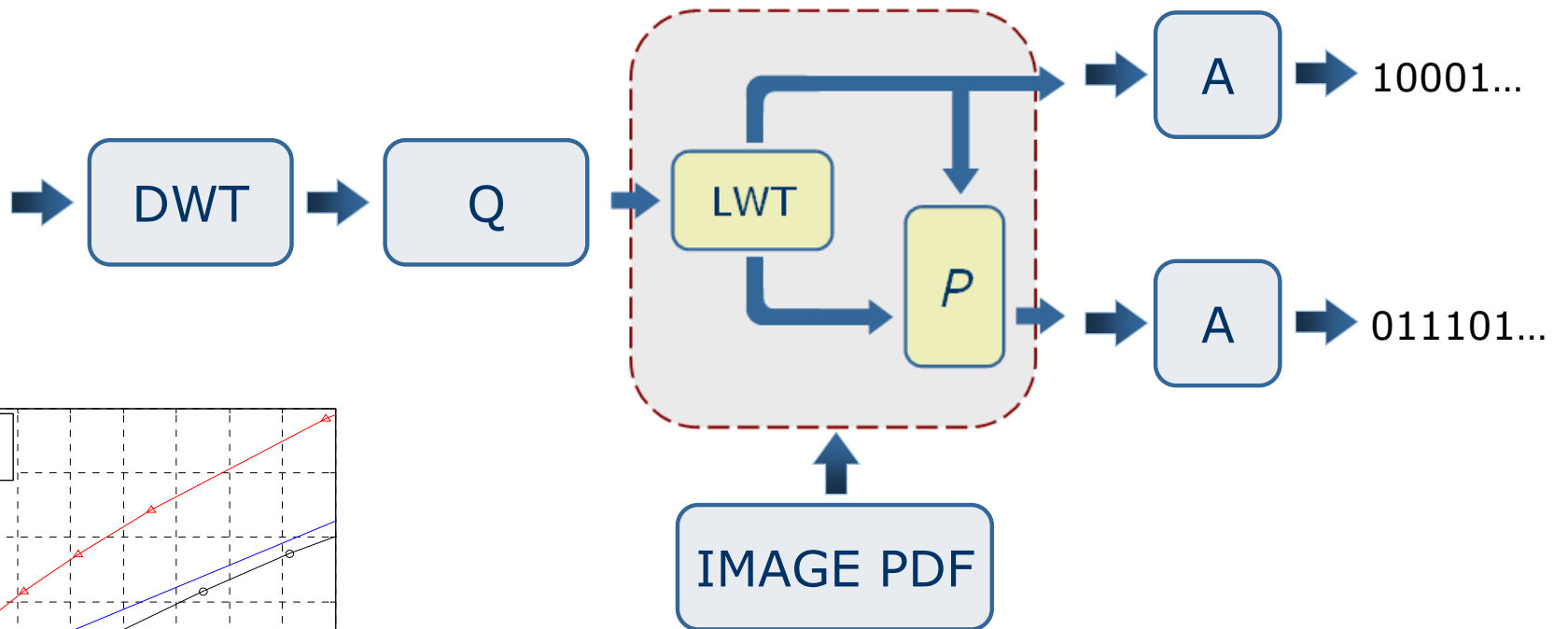
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# Evaluation of the potential: The ideal case

**ASSUMPTION:** THE DECODER KNOWS COMPLETELY THE PDF OF THE IMAGE

ORIGINAL



**LARGE POTENTIAL CODING GAIN**

# Evaluation of the potential: The ideal case

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**IDEAL CASE**

**POTENTIAL FOR THE METHOD  
IDENTIFIED**

**UNREALISTIC TO TRANSMIT  
THE IMAGE PDF**

# Outline

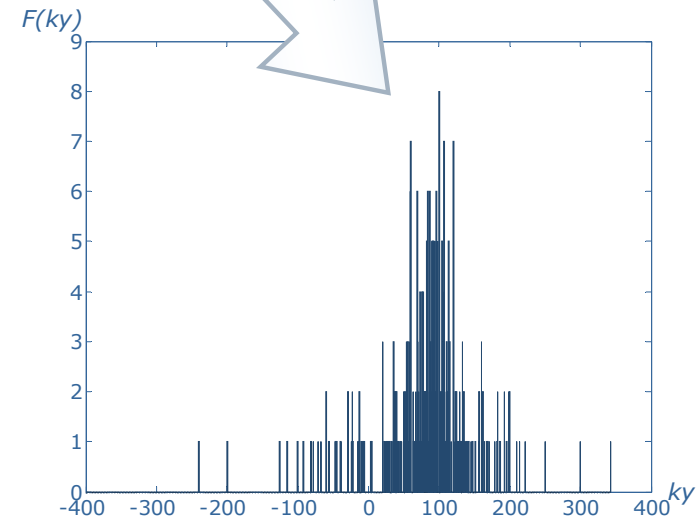
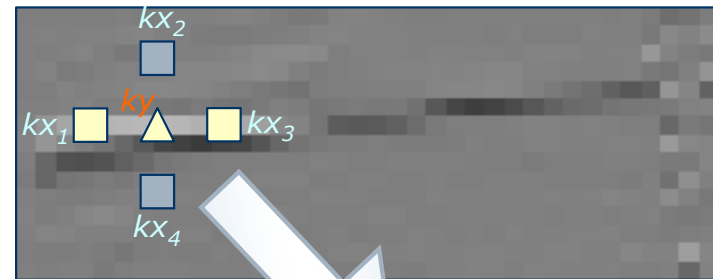
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# Case I. Modeling of contours

- ✓ The idea is to cluster contexts according to their structure in a contrast invariant way
- ✓ We group contexts into classes which produce largest reduction in energy
- ✓ We create a set of models with similar behavior
- ✓ The structures of these models properly describe contours in wavelet domain, where most of the energy is concentrated



# Case I. Modeling of contours

## ✓ Mapping design

- Context models:

1.  $\{A,0,A,0\} \rightarrow A$
2.  $\{A,-A,A,-A\} \rightarrow A$
3.  $\{A,0,A,A\} \rightarrow A$
4.  $\{A,A,A,0\} \rightarrow A$

- Contrast invariant:

$$\{kx_1, kx_2, kx_3, kx_4\} \rightarrow ky$$

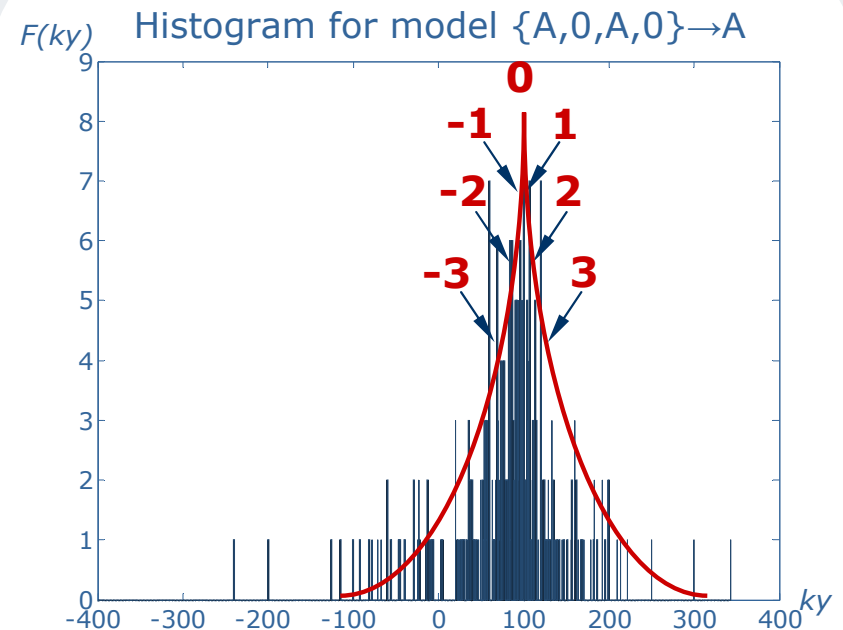
## ✓ Decision for classification

- Distance measure:

$$\begin{aligned} mse &= (kx_1 - A)^2 + (kx_2 - A)^2 \\ &\quad + (kx_3 - A)^2 + (kx_4 - A)^2 \end{aligned}$$

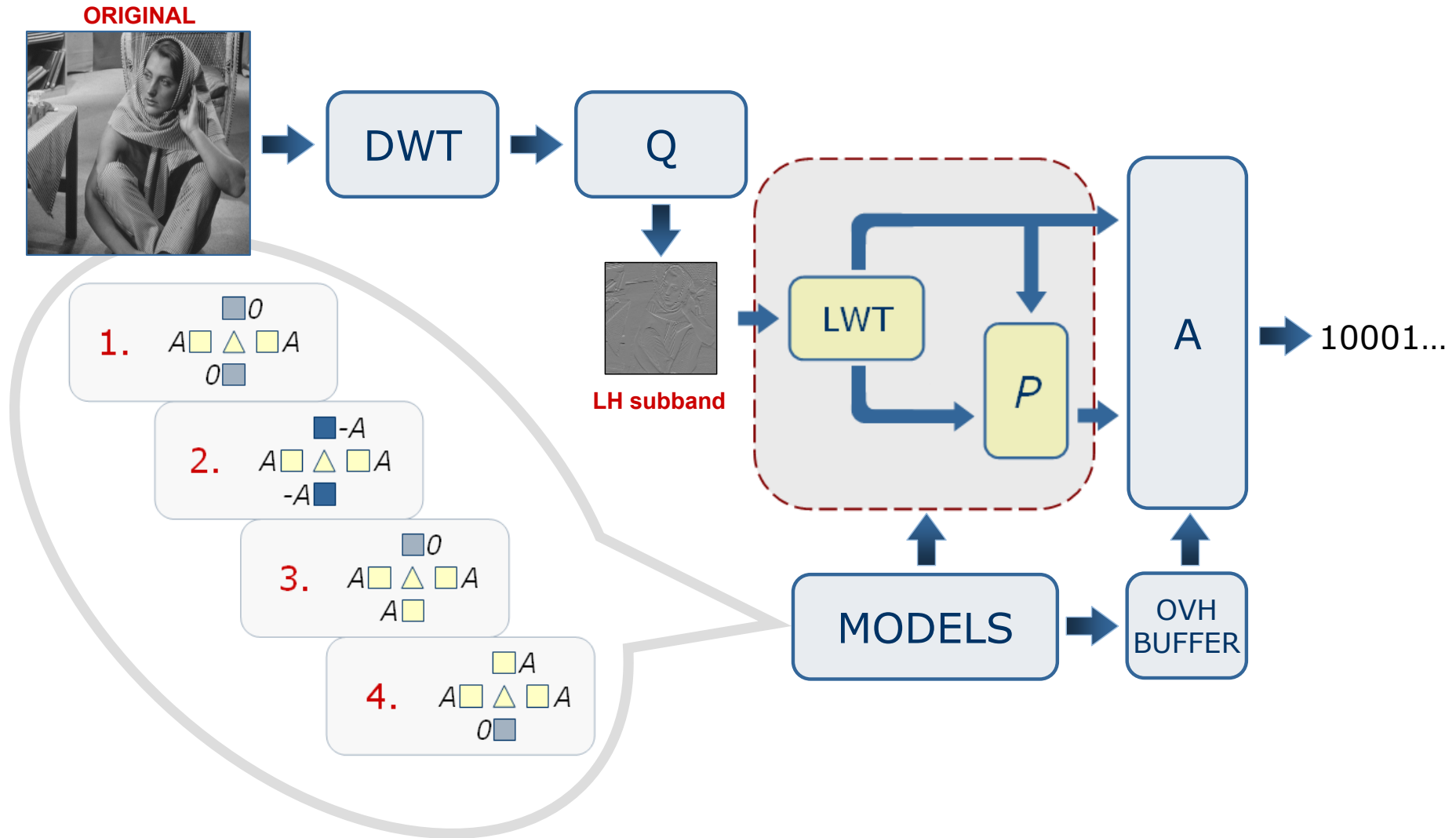
## ✓ Overhead (per model)

$$A, T_{mse}$$



**Mapping is:**  $y' = y - A/k$

# Case I. Modeling of contours

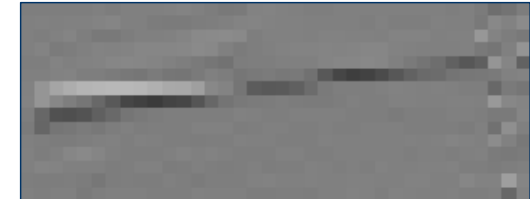


# Case I. Modeling of contours

## EXPERIMENTAL RESULTS

Model	<i>barbara</i>		<i>boat</i>	
	<i>Cont (%)</i>	<i>E<sub>gain</sub> (%)</i>	<i>Cont (%)</i>	<i>E<sub>gain</sub> (%)</i>
1. {A,0,A,0} → A	7.89	<b>-69.96</b>	7.07	<b>-83.69</b>
2. {A,-A,A,-A} → A	16.43	-51.23	21.42	-35.71
3. {A,0,A,A} → A	13.48	-23.37	10.95	7.70
4. {A,A,A,0} → A	10.43	-14.99	9.29	-5.71
Z. {0,0,0,0} → 0	1.7	–	0.93	–
All other contexts	50.09	–	50.34	–

**DWT**



**GL-1**



**GL-2**



Image	Bitrate (bpp)		<i>Coding gain (%)</i>	<i>E<sub>gain</sub> (%)</i>
	DWT	GL		
<i>barbara</i>	4.7755	4.6859	1.91	-36.34
<i>boat</i>	5.2432	5.2224	0.40	-37.50

# Case I. Modeling of contours

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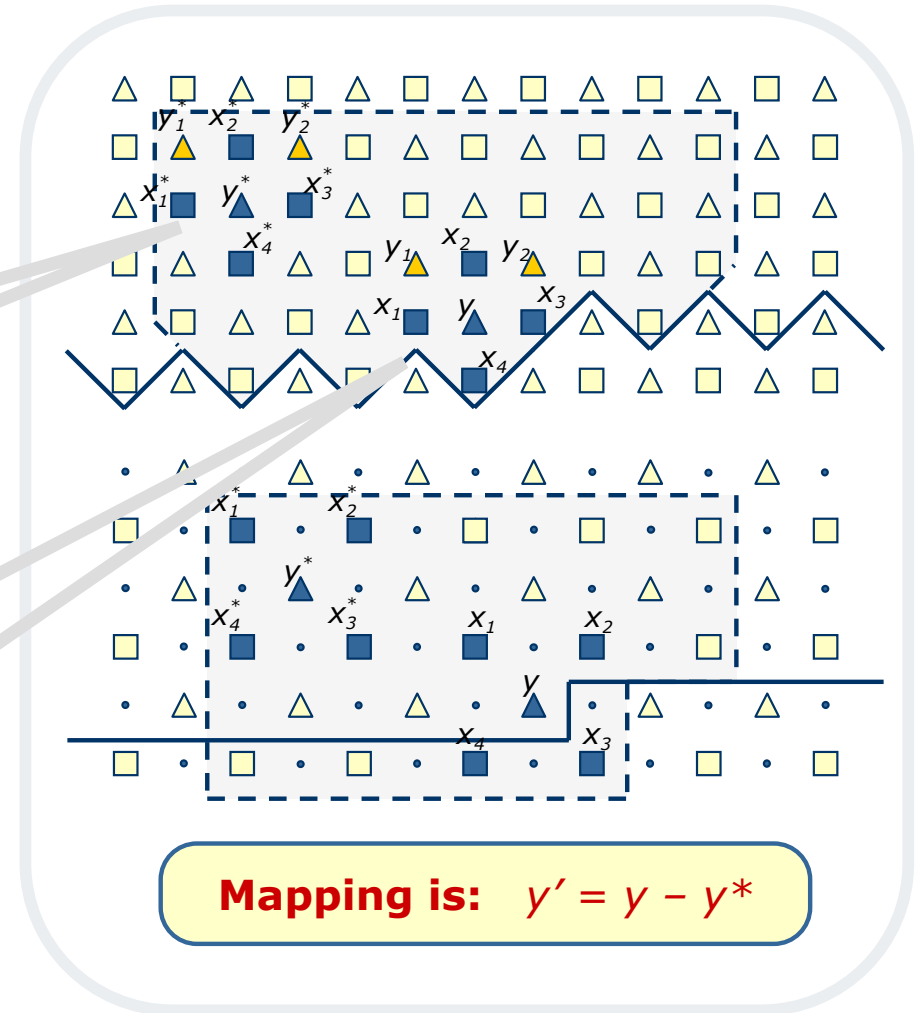
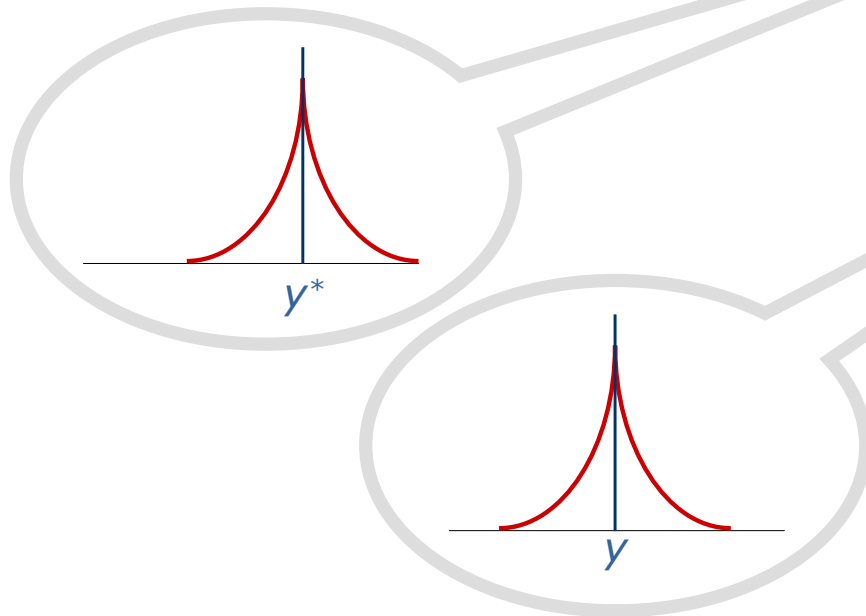
**GENERIC IMAGE CASE I**

**GLOBAL PDF ESTIMATED  
FOR THE MODELS**

**INTERESTING  
POTENTIAL OF THE METHOD  
IN GENERIC CASE I**

# Case II. Adaptive local pdf estimation

- ✓ We find the most similar context w.r.t. the context being encoded, within a neighborhood



# Case II. Adaptive local pdf estimation

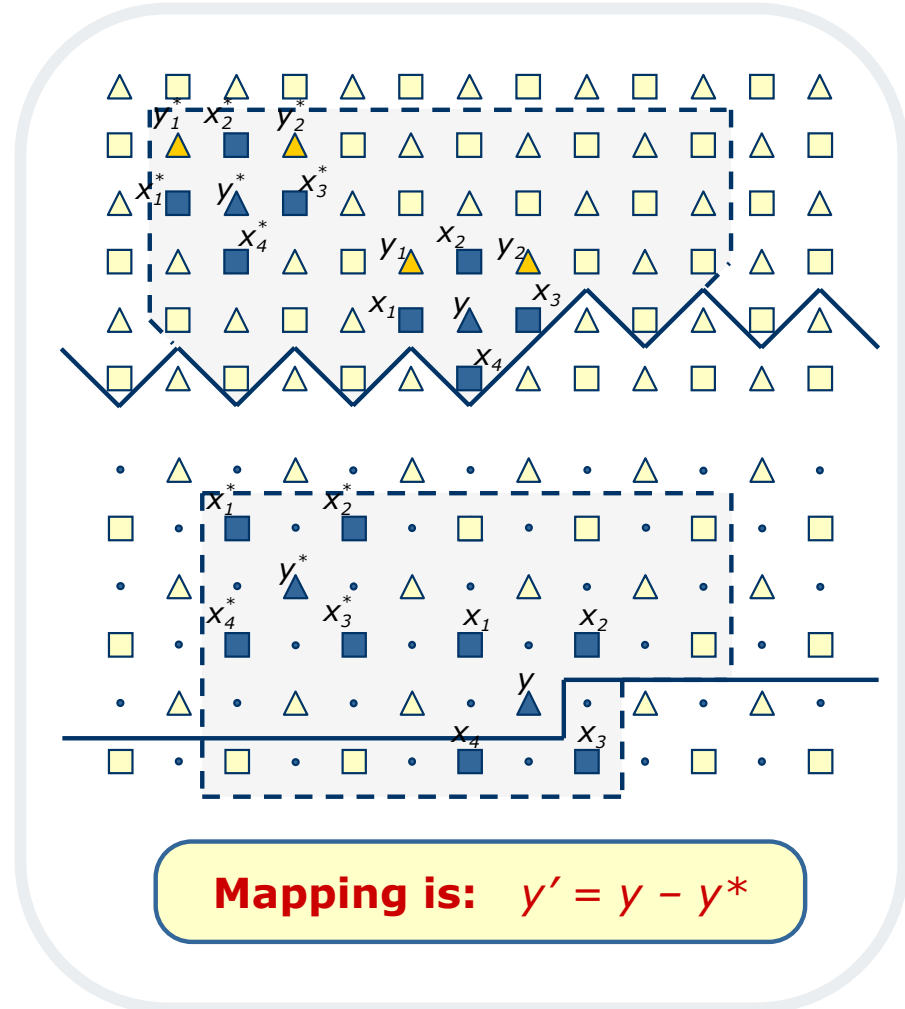
✓ We find the most similar context w.r.t. the context being encoded, within a neighborhood

✓ Distance measure

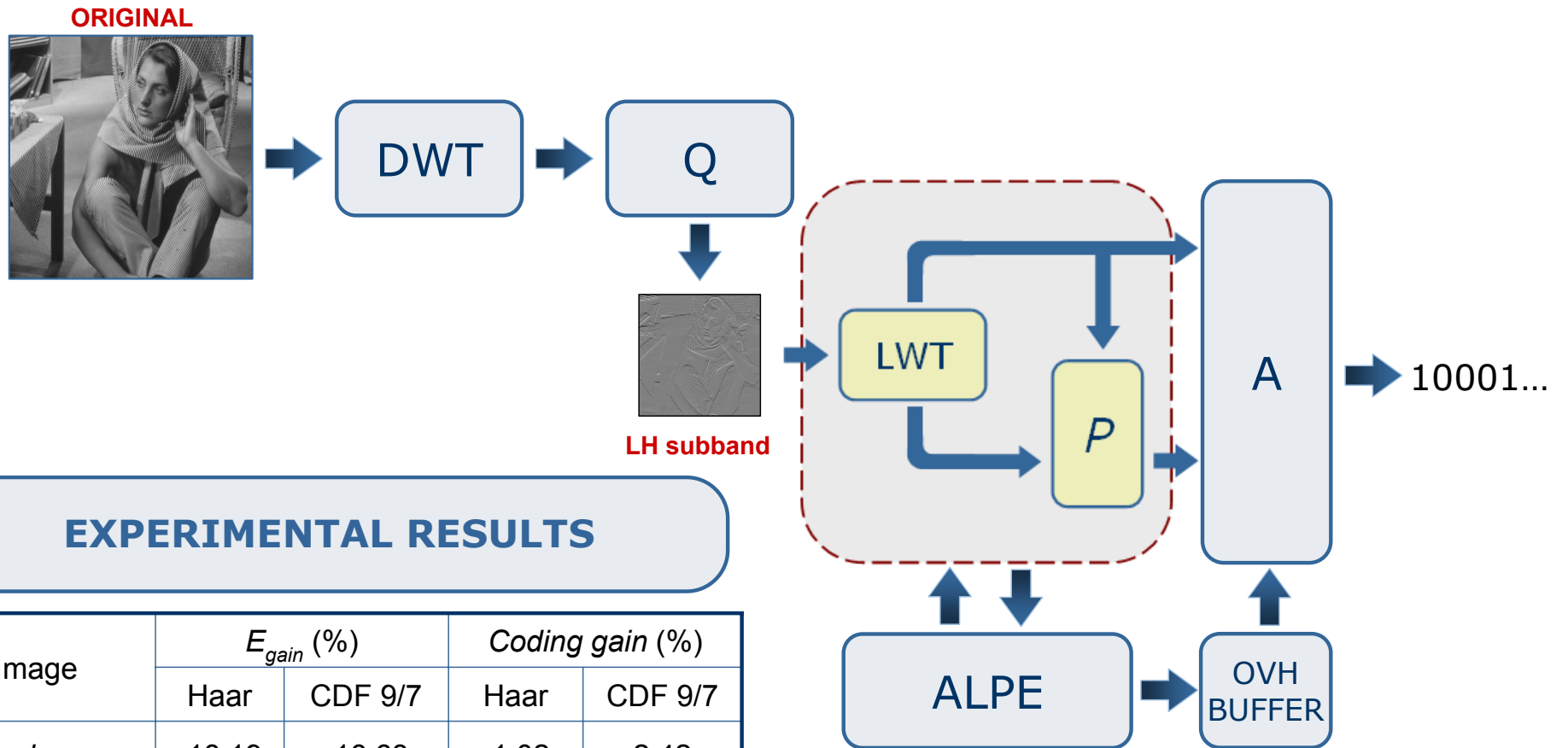
$$D = \frac{\sum_{i=1}^4 (x_i - x_i^*)^2 + \alpha \sum_{i=1}^2 (y_i - y_i^*)^2}{\sum_{i=1}^4 x_i^2 + \alpha \sum_{i=1}^2 y_i^2}$$

✓ Overhead

*window size,  $T_D$ ,  $\alpha$*



# Case II. Adaptive local pdf estimation



## EXPERIMENTAL RESULTS

Image	$E_{gain}$ (%)		Coding gain (%)	
	Haar	CDF 9/7	Haar	CDF 9/7
<i>barbara</i>	-18.19	-16.63	-1.02	2.42
<i>boat</i>	-11.42	-5.12	-0.30	-0.12
<i>bike</i>	<b>-29.25</b>	<b>-28.96</b>	-3.27	-4.57

(ALPE: Adaptive Local Pdf Estimation)



## Case II. Adaptive local pdf estimation

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**GENERIC IMAGE CASE II**

**LOCAL PDF ESTIMATED  
ADAPTIVELY**

**INTERESTING  
POTENTIAL OF THE METHOD  
IN GENERIC CASE II**

# Outline

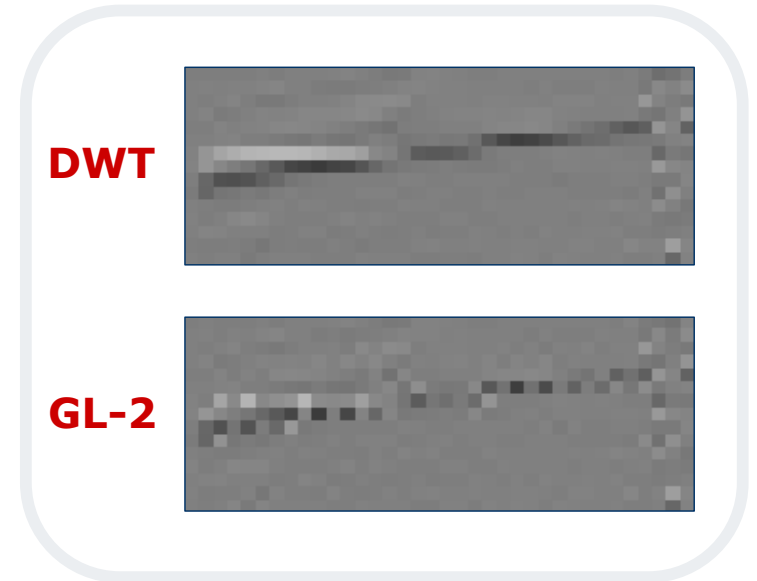
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# Discussion: Entropy coding

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- ✓ Energy minimization GL mapping increases the decorrelation of DWT coefficients
- ✓ This increased decorrelation prevents arithmetic encoder from increasing coding gain significantly
- ✓ The adverse effect of decorrelation is observed in conventional arithmetic encoders, hierarchical coders like SPIHT or SPECK, and in block coders like EBCOT
- ✓ Current work is focused on solving the entropy coding stage



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# Conclusions

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- ✓ The potential of GL method for lossy image coding has been demonstrated in the ideal and the generic cases
- ✓ The energy reduction achieved in the generic cases is considerable, but translates to a moderate reduction in bitrate. At this moment, it does not reflect the expected coding gain
- ✓ Our efforts are now concentrated on the entropy coder:  
your ideas are welcome!

# Generalized Lifting for Lossy Image Coding

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*Xavier Alameda,*  
*Antonio Ortega,*  
*and*  
*Eduardo Mendonça*

for their contributions to this work

Julio Rolón, Philippe Salembier

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Thank you !!!

Julio Rolón, Philippe Salembier

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